is also an expression for that rate;

$$\therefore \qquad du + 2\tfrac{1}{2} \times 2dx^2 = 2xdx + 5dx^2;$$

whence,

$$du = 2xdx.$$

III. Let $u = xy$; then

$$\begin{aligned} u' - u &= xdy + ydx + \ dxdy; \\ u'' - u' &= xdy + ydx + 3dxdy; \\ u''' - u'' &= xdy + ydx + 5dxdy; \\ u'''' - u''' &= xdy + ydx + 7dxdy; \end{aligned}$$

. . . .

In this case the constant difference in the increments of u is $2dxdy$; and it may be shown, by a process of reasoning entirely similar to that used in II, that

$$xdy + ydx + dxdy$$

is the rate at which u increases at the middle of the first unit of time, and that

$$du + dxdy$$

is also an expression for that rate;

$$\therefore \qquad du + dxdy = xdy + ydx + dxdy;$$

whence,

$$du = xdy + ydx.$$

If we put $y = x^2$, we have

$$du = d(xy) = d(x^3) = xd(x^2) + x^2dx.$$

But it has been shown that $d(x^2) = 2xdx$;

$$\therefore \qquad du = d(x^3) = x \times 2xdx + x^2dx = 2x^2dx + x^2dx = 3x^2dx.$$

In this way it may be shown that, if n be any positive integer,

$$du = d(x^n) = nx^{n-1}dx.$$

It may now be shown, in the usual way, that this equation is true, whether n be positive or negative, integral or fractional.

THE METEOR OF AUGUST 11, 1878.

BY PROFESSOR DANIEL KIRKWOOD.

1. *Observations at Bloomington, Ind.*—A few minutes after 10 o'clock* on Sunday evening, Aug. 11, 1878, Rev. John A. Bower of Bloomington, Ind., saw a brilliant meteor near the eastern horizon. Mr. B. had just taken

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*The *precise* time was not noted.

a position facing an open eastern window. The meteor became visible very nearly east of Bloomington—perhaps a few degrees south of east—and about ten degrees above the horizon. Its motion was from south to north. The length of its apparent track was 20° or 25°. The first half of its course was but slightly inclined to the earth's surface; the inclination, however, became sensibly greater towards the point of disappearance, which was N. about 70° E., and very near the horizon. The apparent diameter of the meteor was at least one-third that of the moon. The motion was extremely rapid; the time of flight not exceeding two seconds. No detonation was heard, nor did the meteor separate into fragments at the time of disappearance.

The observations of Mr. Bower were given me verbally. To verify their accuracy I placed myself in the position which he occupied, and had him point out the meteor's course as he had seen it. The foregoing statement, I am satisfied, must be very nearly correct, except as to the time of flight, which is admitted by the observer to be very uncertain.

2. *Titusville, Crawford County, Pennsylvania.*—The following telegram to the associated press appeared in the papers of Tuesday morning, August 13:

"TITUSVILLE, August 12.—A beautiful meteoric display was witnessed from here last evening. The meteor made its appearance in the west at 10:30, moving in a northerly direction. It was of a greenish color and shone with great brilliancy, lighting up the entire surrounding country with a light that for the time prevailed over that of the full moon. Its appearance was only momentary, when it burst and divided into three fragments, two of which assumed a reddish color. Calculating from the time the xplosion was seen until it was heard, the meteor was about 25 miles distant."

3. Oil City, *Venango County, Pennsylvania.*—The papers of the same date contained also the following:

"Oil City, August 12.—A meteor of unusual brilliancy passed here last evening a few minutes after 10 o'clock. It was nearly twice the size of a cannon ball. Its course was north."

All accounts agree that the meteor's course was nearly north. It was seen somewhat west of Titusville; and as the final explosion occurred about 25 miles from that city we may conclude that the track terminated over Crawford County, Pennsylvania. The observations at Bloomington, Ind., indicate that the body first became visible over Western Virginia. The distance directly east from Bloomington to the meridian which bounds Venango county Pa., on the west is 348 miles. Hence when first seen the meteor's altitude was about 77 miles. The length ef the visible track was between

170 and 180 miles—say 175. The most probable velocity therefore decidedly indicates a hyperbolic orbit.

Owing perhaps to the late hour at which the meteor appeared but few observations of the phenomenon were reported. Several letters of inquiry brought no available response, and Prof. Cleveland Abbe of the U. S. Signal Service informed me that no accounts of the meteor were received at the Washington Office.

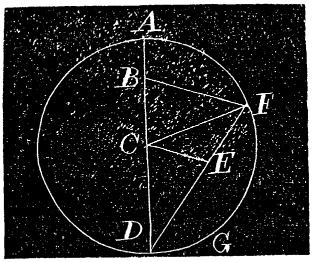
Bloomington, Indiana, Sept., 1878.

QUINQUISECTION OF THE CIRCUMFERENCE OF A CIRCLE.

BY PROF. L. G. BARBOUR, RICHMOND, KENTUCKY.

Theorem.—Let C be the centre of a circle; AD, a diameter. Divide AC in extreme and mean ratio, putting the larger segment next the centre. Then from D as a centre, with a radius equal to DB, describe an arc cutting the circumference in F.

The arc AF will be one fifth of the circumference.



Demonstration.—Join BF, CF and DF and draw CE parallel to BF

By hypothesis

$$AB : BC :: BC : AC.$$

By composition, because $AC = CD$,

$$CD : BC :: BD : CD;$$

$$\therefore \quad BC : CD :: CD : BD.$$

The triangles BCF and CFD, having the same altitude, are to each other as their bases; $\therefore$ $BFC : CFD :: BC : CD$. Similarly

$$CFD : BFD :: CD : BD.$$

But the last couplets of these proportions, themselves form a proportion;

$$\therefore \quad BFC : CFD :: CFD : BFD.$$

Also, since CE is parallel to BF, we have

$$DCE : CFD :: CFD : BFD.$$

Comparing this with the last proportion, we find $BFC = DCE$. These two triangles then are equal in area; the base DE of the one, is equal to the base CF of the other, for CED is isosceles, because similar to BFD, and hence $DE = CD$. Moreover, the angle DCE opposite DE is equal to the angle DBF. But any two triangles of equal areas, equal bases, and having the angles opposite the bases, equal, each to each, are equal in all